



PREDICTORS OF CONCEPT FORMATION AND EDUCATIONAL CONTEXTUALIZATION COMPETENCIES IN GEOMETRY OF PRE-SERVICE MATHEMATICS TEACHERS

Judel V. Protacio

Capiz State University, Philippines

E-mail: jvprotacio@capsu.edu.ph

Abstract

As an interdisciplinary and multicultural educational framework takes the centrality of the delivery of 21st-century education across all levels, this study attempts to address the competencies that prospective and neophyte teachers need to possess to effectively design contextualized education. Through stepwise multiple regression analysis, mathematical communication and mathematical reasoning skills were explored as potential predictors of both concept formation and educational contextualization abilities of 114 randomly selected pre-service mathematics teachers (PMTs) enrolled in their last term of teacher education training in eight Teacher Education Institutions (TEIs) in Panay Island of Western Visayas, Philippines. Results placed the PMTs at the initial stage of developing proficiency in all these competencies, implying learning gaps and skills development deficiencies. Analysis revealed that mathematical communication significantly predicted the concept formation performance, while concept formation ability was likewise a significant predictor of educational contextualization competence. Findings entail that prospective mathematics teachers who demonstrate effective mathematical communication skills also embody profound sense-making and knowledge formation abilities, which in turn contribute to their capability in designing contextualized mathematics education. In enabling policies on curricular directions among TEIs, the study findings offer vantage perspectives towards integrative teaching approaches that curriculum planners and educators may introduce in teacher education training of PMTs.
Keywords: mathematics education, integrative teaching, pre-service teacher education, contextualizing geometry education, mathematical communication, mathematical reasoning

Introduction

The integrative educational framework that spans interdisciplinary and multicultural pedagogical approaches in almost all levels of education requires appropriate teaching techniques and proficient academic personnel to translate instruction into tangible learning. Studies have shown that the specific teaching strategies these approaches entail were found effective in facilitating authentic learning. However, there remain less explored areas on what skillset aspiring teachers need to acquire and demonstrate to meet the pedagogical requirement of delivering contextualized and integrative education (Moely & Connolly, 2020; Roslan et al., 2023). Integrative teaching approaches are strategized by situating the educative process in meaningful contexts; generating operational knowledge through conceptualizing and applying knowledge through problem-centering (Nikitina, 2006). Educational contextualization is founded on the idea that learning is an outcome of employing appropriate contexts in the teaching process (Edwards, 2009). It is a process that situates teaching and learning materials and processes into the interplay of culture, time, and personal experience as a form of humanization of knowledge. This allows the faculty and students make connections among disciplines, personal and practical issues; develops learner agency in learning; and fosters

motivation among learners to know, understand, and appreciate cultural heritage (Benneth et al., 2007; Gebre & Polman, 2020). Consequently, contextualization provides dynamic relationship and continuity between the students' schooling experience and everyday life where students' everyday knowledge become cultural and learning resources (Gilje & Erstad, 2017) in order to achieve conditions for effective learning characterized by deeper conceptual understanding, active student engagement, and skills transfer; hence, achieving higher academic achievements, student retention and progression to advanced studies (Perin, 2011).

Contextualization at the teaching-learning level rests on pedagogical competence of teachers in designing integrative learning experience. This entails continued professional development for in-service teachers and a review of policies and practices in pre-service teacher education programs. At the pre-service stage, there is a need to underscore the role of contextualization as a pedagogical content knowledge to be taught and as a teaching strategy to be employed in learning content at the pre-service training stage.

Concerned with how future mathematics teachers were prepared globally, the International Association for the Evaluation of Educational Achievement (IEA) began in 2007-2008 a comprehensive study to investigate and understand worldwide teacher education training in terms of what beginning mathematics teachers know and can do and how national policies in countries contribute to pre-service teacher education outcomes (Tatto et al., 2008, Tatto, 2020). The study measured content knowledge (CK) and pedagogical content knowledge (PCK) of PMTs in 21 countries. The study observed that future teachers with deeper mathematical understanding were more capable of interpreting students' thinking and making appropriate response (Tatto et al., 2012). This shows that PMTs with profound knowledge of content have better chances of effectively teaching the content. Hence, establishing provisions towards sufficient cognitive development and the inclusion of interdisciplinary and multicultural teaching approaches in the curriculum may guarantee an effective training of future mathematics teachers.

Contextualization in Mathematics Education

Mathematics in education can be viewed as a cultural construction that draws from familiar contexts - experiences, environments and processes the development of cognitive and problem-solving strategies, and achievement of substantial sense-making and knowledge formation (Chavarria-Arroyo & Albanse, 2023; Mallart et al., 2018). Contextualization in education began towards the later part of the 20th century, spanning more than three decades of inquiry and practice in education (Gilbert, 2006; Hofstein et al., 2011; Silzeth & Erstad, 2018). Studies on implementing contextualized education in mathematics and science education have shown its potential to affect learning across learning levels (Nashon & Anderson, 2013; Nilson & Ryve, 2014; Parr et al., 2008; Picardial & Sanchez, 2022; Valdez & Bungihan, 2019). Furthermore, contextualization through real mathematics education (RME) was found to enhance other higher order cognitive abilities such as mathematical reasoning and mathematical communication skills of students (Palinusa et al., 2021).

Despite consistent studies on the efficacy of contextualization in education, several gaps in theory and practice were noted. These include issues on identification of contexts and how they are appropriately framed in various learning environments; most contexts defined by teachers often limit student agency in the learning process (Gebre & Polman, 2020); development of assessment procedures that incorporate basic skills and content knowledge to determine the effects of contextualization; and the amount of contextualization to be integrated in a learning activity (Perin, 2011). In view of teacher education pre-service training, there is a lack of research that looked into how contextualization is integrated into the teacher education curriculum and the necessary cognitive skills that contribute to meaningful design of contextualized teaching procedures by future teachers. Hence, this study sought to establish an

anchor point for contextualized pre-service training in one of the content courses in mathematics education.

Concept Formation and Inquiry

Concept formation is geared towards knowledge formation. It is a teaching-learning dimension that is viewed as an individual cognitive ability to operationalize ideas on the one hand, and as a teaching strategy on the other. Vygotsky's theory of concept formation provides a strong framework on the role of language in concept formation. According to Vygotsky (1986, p. 106), "It is a functional use of the word, or any other sign, as a means of focusing one's attention, selecting distinctive features and analyzing and synthesizing them, that plays a central role in concept formation."

As concepts are initially developed by individuals, words are used as both mediators and concept representations. The individual's interaction with the social world through communication enables him to reconcile his personal concept with that of the external world, thus, the developed concept becomes cognitively and socially constituted (Radford, 2000). As a learning ability, it involves higher-order mental processes on how individuals develop an empirical notion or operational knowledge through a sequence of cognitive processes (Pielsticker, 2021; Rao, 2006; Schunk, 2012) that include exploration, comparison, classification, generalization, and abstraction (see Marzano, 2000) as used in reasoning and inquiry.

As an integrative teaching strategy, concept formation is characterized as basic science. Thus, it is a scientific method-oriented approach and plays an essential role in mathematical investigation, discovery learning, and scientific inquiries (Nikitina 2006). Successful concept formation in inquiry requires that learning is an active process through exploration and development of empirical or operational concepts by formulating hypotheses, testing them, and connecting them to other fields of application (Burscheid & Struve, 2020; Pielsticker, 2021). Consequently, learners through inquiry achieve profound knowledge and understanding or cognitive gains, become engaged and motivated in the search for new knowledge, develop agency in learning, and transfer newly learned knowledge and skills to different contexts (Duran & Dokme, 2016; Eshetu et al., 2022; Laursen et al., 2014; Teig, 2021; Teig et al., 2018).

With the emphasis of 21st-century education on student-centered lifelong learning, educational systems now take inquiry as an essential component of curriculum delivery (Chu et al., 2017a, 2017b; Fuja & Asrizal, 2021; Preston et al., 2015). While decades of research and practice have shown the efficacy of discovery and inquiry-based learning approaches in achieving desired learning outcomes (Duran & Dokme, 2016; Jerrim et al., 2019; Teig, 2021) there are prevailing implementation challenges noted to include teachers' lack of experience and inadequate pedagogical content knowledge on inquiry-based teaching strategies, high time and resources requirements for schools and teachers, and the demand for students to do high level of independent learning (Moely & Connolly, 2020; Roslan et al., 2023). While inquiry has been widely acclaimed as a successful teaching approach mostly in science education, there remains enough room for its implementation in other areas such as mathematics and social sciences (Caswell & Derek, 2017; Moely & Connolly, 2020).

Mathematical Communication and Reasoning Skills

Mathematics is a unique language made up of words, illustrations, tables, graphs, and symbols forming expressions and equations (Tong et al., 2021) that are embedded within another language as the medium (i.e., mathematics in English or mother tongue). To communicate mathematically is to use mathematical language in expressing mathematical ideas (Kamid et al., 2020). Thus, a hallmark of mathematics education is for learners to develop effective

mathematical communication skills (Tong et al., 2021). Recent literature connects mathematical communication with other learning and cognitive abilities, teaching approaches, and the nature of learners. Tong et al. (2021) characterized mathematical communication as the ability to identify, understand and solve mathematical problems expressed in mathematical language; accurately and logically state, explain and prove mathematical concepts and specify applications of mathematical concepts. Mathematical communication skills are also enhanced through guided discovery and problem-based learning models among middle high school students (Supriyanto et al., 2020; Wandari & Fardillah, 2020; Supriyanto et al., 2020; Zulkarnain et al., 2021), contextualized instruction such as Realistic Mathematics Education (RME) (Andriani & Fauzan, 2019; Hutapea et al., 2019; Palinussa, 2021). As for cognitive style, communication skills vary between field-independent and field-dependent upper secondary learners (Kamid et al., 2020).

The characterization of mathematical communication skills by Tong (2021) suggests a more fundamental skill that is contained in the process of accurately and logically presenting, explaining and proving mathematical ideas—mathematical reasoning skill. This skill is demonstrated when students explain their notions, make comparisons and inferences, justify deductions and decisions, and apply and transfer learning from one context to another (Australian Curriculum, 2013; Mata-Pereira et al., 2017). With the centrality of mathematical reasoning in mathematics learning, an enhanced mathematical reasoning is essential towards achieving mathematical fluency, particularly in the understanding of mathematical concepts and their applications in various contexts (Mukuka et al., 2023). However, Mukuka et al. (2019) showed that mathematical reasoning among pre-service teachers is a less developed ability, where they demonstrated limited conceptual understanding of mathematical proof. Similarly, there is evidence of discrepancy between the in-service teachers' claims on supporting reasoning skills development and the actual learning opportunities they provide to promote communication and reasoning skills (Mukuka et al. 2021). As such, falling short on optimizing the provision of relevant learning experience reflects the need for pre-service and in-service teachers to be equipped with appropriate teaching strategies that promote higher order cognitive skills such as mathematical reasoning and mathematical communication.

Research Aim

This study looked into the learning achievements of PMTs in mathematical communication and mathematical reasoning and how these competencies influence their operationalization of mathematical concepts and utilization of contextualization in teaching. The assumptions of constructivism and socio-cognitive learning theory, which posit that learning is constructed by students through social contexts (Schunk, 2012), provide the theoretical framework for the study. Consequently, the pre-service teachers' cognitive and pedagogical abilities and how they utilize these abilities to facilitate the attainment of learning outcomes are assumed to reflect their pre-service teacher education experience. Hence, the pre-service teacher education curriculum has to provide relevant learning experience that addresses the development of cognitive and pedagogical capabilities of PMTs to answer the demands of their future workplace. In view of these concerns, the study sought to answer the following research questions:

1. What is the extent of mathematical communication, mathematical reasoning, concept formation, and educational contextualization competencies of PMTs?
2. Are there significant correlations among these competencies?
3. Which of the competencies are predictors of concept formation and educational contextualization skills of PMTS?

Research Methodology

Design

This quantitative research employed a cross-sectional survey-predictive correlational design intended to investigate the extent of development and relationships among the identified competencies of PMTs. According to Frankael et al. (2012), a cross-sectional survey gathers data through asking questions to members of a sample drawn from a predetermined population with the purpose of describing characteristics of the population from which the sample was drawn. In this study, data were gathered from randomly selected participants from a finite population of PMTs through researcher-developed instruments personally administered by the researcher. As predictive correlational research, which ascertains relationships between variables and predicts future outcomes without variable manipulation (Creswell & Creswell, 2017; Keith, 2019), this study utilized predictive correlation analysis to determine pairwise relationships among the competencies, and to identify significant predictors of concept formation and educational contextualization skills. Findings were interpreted within the assumptions of quantitative analysis and the theoretical framework of the study.

Sample

A total of 114 graduating PMTs participated in the study. These PMTs were enrolled in eight TEIs in Panay Island of the Western Visayas region in the Philippines. As they were in the last term of their teacher education training, they were deployed as practice teachers in basic education schools in compliance with the 360-hour teaching internship requirement. During data collection, the respondents were in their last month of their internship; hence, they had tangible experience of supervised actual classroom teaching.

The sample size was calculated from the population of 161 PMTs in the same year level using Cochran's formula at margin of error $e = .05$ and population percentage $p = .5$. With three predictor variables, this sample size sufficed for the required minimum of 15 participants per predictor set by Stevens (1996) for multiple regression analysis in social sciences or the criteria $N > 50 + 8m$ where N is the total sample size and m is the number of predictors by Tabachnik and Fidell (2007). Stratified sampling was used to proportionally allocate the respondents to their TEIs, and simple random sampling was then used to select PMTs from their respective institutions. Table 1 below shows the distribution of the respondents by institution and gender. The highest number of participants was 30(26.32%) coming from two TEIs, while the least number was 3(2.63%). Notably, about 7 out of 10 participants in the study were female. They were between 21 and 22 years old. The participation of PMTs in the study, including those in the pilot testing of instruments, was voluntary, with each executing informed consent.

Table 1
Distribution of Respondents by Institution and Gender

Sociodemographic characteristic	<i>n</i>	%
Institution		
Teacher Education Institution 1	30	26.32
Teacher Education Institution 2	3	2.63
Teacher Education Institution 3	30	26.32
Teacher Education Institution 4	4	3.51
Teacher Education Institution 5	21	18.42
Teacher Education Institution 6	7	6.14
Teacher Education Institution 7	13	11.40
Teacher Education Institution 8	6	5.26
Gender		
Male	46	30.35
Female	68	69.65

Instruments and Procedures

The study utilized four sets of validated and pilot-tested researcher-developed instruments described in the following sections. Content and form validation was conducted by five experts in mathematics education who were actively engaged in pre-service mathematics teacher education training in their respective TEIs. These educators have doctoral degrees in mathematics education. After validation and revisions, the final drafts were pilot-tested on 25 PMTs enrolled in the university where the author was affiliated. Scoring rubrics were developed and subjected to validation along with the competency tests. The test-retest method in the pilot testing was used with a one-week interval between the testing periods. The reliability of each instrument was determined by correlating the two sets of scores using Pearson's *r*. With the obtained correlation coefficients greater than .70, the developed instruments were considered to have an acceptable degree of reliability: mathematical communication ($r = .895$), reasoning skills ($r = .895$), concept formation ($r = .939$), and contextualizing instruction ($r = .890$).

Mathematical Communication Skills Test. In designing the research instrument to assess mathematical communication skills, the researcher adapted the constructs characterizing mathematical communication as familiarity with symbols representing mathematical ideas and awareness of the syntactic rules for writing symbols in acceptable form (Bergqvist & Lithner, 2012); accurate and logical presentation, illustration, and proof mathematical ideas (Tong et al., 2021) drawn from adequate mathematical vocabulary (Rustam & Ramlan, 2017). The developed instrument consisted of 35 items to measure the ability of the respondents in written English language in the context of mathematics, particularly on verbal ability - appropriate use of mathematical term or expression to express an idea clearly in formal written communication (10), vocabulary - meaning of a mathematical term, expression or notation (10), identifying and correcting errors in mathematical statements (8), and reading comprehension and proof organization (7). One (1) point is assigned to every correct answer.

Mathematical Reasoning Skills Test. The development of this research instrument is based on the concepts that mathematical reasoning is a higher order ability that is both deductive and logical (Jader et al., 2017; Mata-Pereira et al., 2017), and a chain of premises or deductions that learners adopt to draw a conclusion when completing a mathematics activity which may be both deductive and inductive (Lithner, 2008). Guided by these constructs, the research instrument consisted of 10 mathematical-logical situations that required students to apply mathematical inductive reasoning (3), mathematical deductive reasoning (3), and general logical reasoning (4). A 5-point analytic rubric was used to score responses.

Concept Formation Skills Test. A 10-item research instrument consisting of mathematical situations intended for guided inquiry was used to measure the ability of PSMTs to formulate concepts in geometry. The framework for the development of the instrument is based on the characterization of concept formation in terms of the thinking skills that include (Marzano, 2000): systematic exploration through comparing, classifying, organizing; logical analysis and abstraction, and mathematical concept formation. A 5-point rubric was used to score responses in each of these aspects of conceptualization.

Instructional Contextualization Competence Test. This instrument assessed the ability of PMTs to associate a mathematical concept to different contexts, formulate a contextualized learning activity, and specify a contextualized assessment strategy in assessing learning. The framework used in the development of this instrument is based on the classification of contexts identified by Metz (2013). The resulting instrument consisted of three parts: relating concepts to contexts (4), contextualizing instruction procedures (4), and contextualizing assessments (4). A 5-point rubric was used to evaluate the responses of the PMTs in each of the items.

The instruments were administered by the researcher in two separate sessions in each of the participating TEIs. The instruments on mathematical communication and mathematical reasoning were administered as verbal tests on the first session with one hour allocation for each test. The assessment of concept formation and educational contextualization skills was done in the second session and was administered through a workshop setting with a two-hour allocation for each test. Before the workshop, an orientation session was conducted by the researcher to set the direction in answering the instruments.

Data Analysis

To describe the performance of PMTs in each of the assessed competencies, the mean, standard deviation and percentage score were used. The percentage score is the ratio, expressed in percent, of the mean score to the highest possible score in an item or component of a test. In open-ended tests, rubrics were used to score responses. Content analysis was also used to identify learning gaps and skills development deficiencies in each of the items. A scheme using mean percentage score (MPS) was used to interpret the extent of performance of PMTs and identify learning gaps. It includes: Below 50 (Beginning level), 50.00 – 62.50 (Developing), 62.51 – 75.00 (Approaching Proficiency), 75.01 – 87.50 (Proficient), 87.51 – 100.00 (Advanced).

Analyses of correlations and predictive relationships utilized Pearson's r and stepwise multiple regression analysis at 5% significance level, respectively. To ensure that standards of inferential analysis were observed, normality tests were performed to check on the initial assumptions of parametric correlation analysis. Kolmogorov-Smirnov test results shown in Table 2 indicate that data in each variable are normally distributed ($p > .05$).

Table 2

Kolmogorov-Smirnov Normality Tests on the Distribution of Data in Each Study Variable

Study variables	Statistic	df	p
Mathematical communication	.070	114	.200
Mathematical reasoning	.079	114	.078
Concept formation	.067	114	.200
Educational contextualization	.066	114	.200

Research Results

Extent of Competencies of PMTs

The descriptive analysis of mathematical communication skills of PMTs used mean, standard deviation, and mean percentage score. Results shown in Table 3 indicate that the skills development in this competence was at the beginning level ($M = 13.49$, $SD = 4.42$, $MPS = 38.54$). Content analysis of responses revealed that PMTs manifested a developing level of ability on reading comprehension and proof organization and needed substantial improvement on identifying and correcting errors, verbal ability and mathematics vocabulary. The PMTs appeared to have difficulty in distinguishing expressions that represent equalities and inequalities in statements and graphs; and no clear distinction on the correct use of *always*, *sometimes*, *never* in plane and space contexts. These learning gaps in mathematics vocabulary were due to unfamiliarity with and inability to comprehend the technical terms in mathematics such as function, undefined, uniqueness, and existence; unfamiliarity with meanings and conditions of symbols like the quantifier $\forall \dots \exists$, parallelism $//$, similarity $\sim$, congruence $\cong$, divisibility $|$. In identifying and correcting errors, PMTs showed limited ability in formulating structurally correct and adequate definitions, theorems, and postulates to correct an erroneously framed statement. In terms of reading comprehension and proof organization, future teachers had difficulty in organizing statements to form valid arguments in writing proofs and in applying and analyzing ideas in a reading selection.

Table 3

Extent of Mathematical Communication Skills of PMTs

Mathematical communication skills	Score	M	SD	Mean percentage score
Verbal ability	10	3.88	1.81	38.80
Mathematics vocabulary	10	3.85	1.82	38.50
Identifying and correcting errors	8	1.52	1.23	19.00
Reading comprehension and proof organization	7	2.14	1.56	26.75
Total	35	13.49	4.42	38.54

Note: MPS interpretation is based on the scale: Below 50.00 (Beginning), 50.00 – 62.50 (Developing), 62.51 – 75.00 (Approaching Proficiency), 75.01 – 87.50 (Proficient), 87.51 – 100.00 (Advanced).

The assessment of reasoning skills involved three components, two of which are explicitly mathematical in context: mathematical inductive reasoning which uses specific cases to form a generalization involving mathematical concepts and operation; mathematical deductive reasoning which applies general mathematics principle or concept to a specific case of application; and general logical reasoning which employs semantics, logical thinking or a combination of inductive and deductive reasoning in familiar situations. The overall extent of reasoning skills of PMTs shown in Table 4 was at the beginning level ($M = 15.23$, $SD = 2.58$, $MPS = 30.46$). Their skill in each of the three forms of reasoning was also at the beginning level with deductive reasoning as the least developed, followed by logical reasoning, and then deductive reasoning. Analysis of the responses identified the learning gaps behind low-level performance. In mathematical inductive reasoning, PMTs manifested difficulty in identifying and abstracting patterns into coherent statements as conclusions. In mathematical deductive reasoning, there was an inability to recall and translate appropriate mathematical principles into specific cases; and in non-mathematical logical reasoning, the PMTs had difficulty in applying negation and truth values, making correspondence through elimination, and analyzing semantic clues in arguments.

Table 4
Extent of Mathematical Reasoning Skills of PMTs

Reasoning skills	Score	<i>M</i>	<i>SD</i>	Mean percentage score
Mathematical inductive reasoning	15	7.09	2.47	47.27
Mathematical deductive reasoning	15	2.67	1.25	17.8
General logical reasoning	20	5.58	2.53	27.90
Total	50	15.23	2.58	30.46

Note. MPS interpretation is based on the scale: Below 50.00 (Beginning), 50.00 – 62.50 (Developing), 62.51 – 75.00 (Approaching Proficiency), 75.01 – 87.50 (Proficient), 87.51 – 100.00 (Advanced).

The assessment of concept formation performance involved three aspects that include exploration, abstraction, and concept formulation. Using the mean, standard deviation and mean percentage score, the extent of skill development was analyzed and interpreted. Results show that the PMTs were at the beginning level of their concept formation ability ($M = 50.85$, $SD = 21.67$, $MPS = 33.90$). As reflected in Table 5, their ability to do abstraction from specific cases was the least developed, followed by operational formulation of concepts and exploration of a given situation. Content analysis of the responses identified learning gaps in the concept formation ability of PMTs in terms of their inability in systematically exploring a mathematical situation as a starting point in conducting mathematical investigation. This then led to their difficulty in discovering and abstracting patterns, which consequently yielded erroneously framed concepts from investigation.

Table 5
Extent of Concept Formation Skills of PMTs

Concept formation skills	Score	<i>M</i>	<i>SD</i>	Mean percentage score
Exploration	50	22.62	8.75	45.24
Abstraction	50	14.39	9.67	28.78
Concept formulation	50	20.88	9.39	41.76
Total	150	50.85	21.67	33.90

Note: MPS interpretation is based on the scale: Below 50.00 (Beginning), 50.00 – 62.50 (Developing), 62.51 – 75.00 (Approaching Proficiency), 75.01 – 87.50 (Proficient), 87.51 – 100.00 (Advanced).

In assessing educational contextualization, three aspects of educative process were considered: relating concept to context, contextualizing teaching procedure, and contextualizing assessment. Mean, standard deviation and mean percentage score were used to analyze and interpret the extent of skill achievement in contextualization. Reflected in Table 6, the extent of educational contextualization skill was at the beginning level ($M = 24.41$, $SD = 9.41$, $MPS = 32.55$). Specifically, the least learned skill in contextualizing education was contextualizing assessment, followed by contextualizing teaching procedures and then relating concept to context. In analyzing responses, the PMTs were found to be less proficient in relating the given topics in geometry to familiar or real-life situations, fictional or contrived contexts, related topics in mathematics, and other fields of knowledge.

Table 6
Extent of Educational Contextualization Competence of PMTs

Educational contextualization	Score	<i>M</i>	<i>SD</i>	Mean percentage score
Relating concept to context	25	11.76	4.06	47.04
Contextualizing teaching procedure	25	6.74	4.32	26.96
Contextualizing assessment	25	5.91	4.10	23.64
Total	75	24.41	9.41	32.55

Note: MPS interpretation is based on the scale: Below 50.00 (Beginning), 50.00 – 62.50 (Developing), 62.51 – 75.00 (Approaching Proficiency), 75.01 – 87.50 (Proficient), 87.51 – 100.00 (Advanced)

Correlations among Competencies of PMTs

Results of pairwise correlation analyses involving the identified cognitive and pedagogical abilities of PMTs are reflected in Table 7. Findings showed highly significant, weak, and positive correlations between these variables: mathematical communication and reasoning skills ($r = .275$), mathematical communication and educational contextualization competence ($r = .378$), mathematical reasoning and concept formation skills ($r = .295$), mathematical reasoning and educational contextualization competence ($r = .262$). This shows that while the relationship between the variables in each pair is consistent, the influence of one on the other is mostly minimal. Highly significant, moderate and positive correlations were noted between mathematical communication and concept formation skills ($r = .652$), and concept formation and educational contextualization competence ($r = .479$), suggesting the observable influence of one variable over the other. Furthermore, significant pairwise correlations between these variables indicate the interconnectedness among the identified

higher order cognitive and contextualization abilities of PMTs. As a reference for regression analysis, the correlation coefficients also indicate no multicollinearity among the variables ($r < .70$) and identify potential predictors of concept formation and educational contextualization competencies ($r > .30$).

Table 7
Correlations among the Identified Competencies of PMTs

Study variables		Mathematical communication	Mathematical reasoning	Concept formation
Mathematical communication	r	1.00		
	p	--		
Mathematical reasoning	r	.275		
	p	.002		
Concept formation	r	.652	.295	
	p	<.001	.001	
Educational contextualization	r	.378	.262	.479
	p	<.001	.002	<.001

Predictor of Concept Formation Skills

A stepwise regression analysis was performed to determine significant predictors of the concept formation performance of PMTs. Results presented in Table 8 reveal a highly significant linear relationship between mathematical communication and concept formation skills, . Given the regression statistics in Table 9, the coefficient of determination indicates that mathematical communication accounts for 42.60 % of the variance in concept formation ability. The model yields the regression equation Concept Formation = 7.708 + 3.198 (Mathematical communication) highlighting the significant contribution of mathematical communication in the ability of PMTs to conceptualize mathematical ideas.

Table 8
Analysis of Variance for Mathematical Communication against Concept Formation

ANOVA	df	SS	MS	F	p
Regression	1	22600.401	22600.401	83.035	< .001
Residual	112	30484.163	272.180		
Total	113	53084.564			

Table 9
Stepwise Regression Result for Concept Formation

Variable	B	95% CI for B		SE B	β	R^2	ΔR^2
		LL	UL				
Model 1						.426	.426
Constant	7.708	-2.158	17.575	4.98			
Mathematical communication	3.198***	2.502	3.893	.351	.652		

Note: CI = Class Interval; LL = Lower Limit; UL = Upper Limit

*** $p < .001$

Predictor of Educational Contextualization Competence

To determine significant predictors of educational contextualization competence among concept formation, mathematical communication and reasoning skills, a stepwise multiple regression analysis was used. Results in Table 10 identify concept formation as a significant predictor of educational contextualization competence. The highly significant linear correlation between the two variables affirms the consistency of influence between the two variables. The regression statistics presented in Table 11 indicate that with , about 22.90 % of the variance in the educational contextualization ability of PMTs is due to their concept formation skills. The B coefficients give the regression model Educational contextualization = $13.848 + 0.208$ (Concept formation), which suggests significant and positive contribution of the ability of PMTs to conceptualize mathematical ideas to their competence in the delivery of contextualized mathematics education.

Table 10
Analysis of Variance for Concept Formation against Educational Contextualization

ANOVA	df	SS	MS	F	p
Regression	1	2292.173	2292.173	33.308	< .001
Residual	112	7707.500	68.817		
Total	113	9999.673			

Table 11
Stepwise Regression Result for Educational Contextualization

Variable	B	95% CI for B		SE B	β	R^2	ΔR^2
		LL	UL				
Model 1						.229	.229
Constant	13.848***	9.908	17.789	4.98			
Concept formation	.208***	.136	.279	.036	.479		

Note: CI = Class Interval; LL = Lower Limit; UL = Upper Limit

*** $p < .001$

Discussion

Analyses of the extent of skills development in mathematical communication, mathematical reasoning, concept formation, and educational contextualization of PMTs revealed learning achievements below expectations. The consistent beginning level of abilities of PMTs in these competencies signifies performance that barely reaches the minimum or fundamental level and points out problematic learning and skills development. The assessment of mathematical communication skills revealed a considerable degree of difficulty of PMTs in applying requisite cognitive strategies, such as analyzing the specific use of technical language of mathematics and comprehension in ascertaining the accuracy of a statement on a mathematical concept or principle. In this study, it can be noted that the English language used as a medium for teaching and learning of mathematics was a second language among PMTs. As mathematics itself is a language of its own, non-native speakers of the medium may experience two aspects of language barriers which counter learning acquisition and transfer. Since language facility is integral in mathematics learning (Bailey et al., 2016; Kleemans & Segers, 2020; Sigley & Wilkinson, 2015; Vukovic & Lesaux, 2013), focused treatment on language structure and vocabulary, as well as on the oral and written form of language expression is imperative in doing mathematics for PMTs.

The low-level achievement of mathematical reasoning skills in the areas of inductive, deductive and general logical reasoning affirms the inability of PMTs to navigate from specific to general cases of reasoning and vice versa, and to apply mathematical logic to an implicitly framed mathematical situation. This finding verifies similar research findings revealing that students' reasoning ability was imitative and routinary, often experienced in traditional teaching settings (Sukirawan et al., 2018) and inadequately developed on making deductions, assessing argument validation, and formulating the generalization (Mukuka, 2021; Novienda & Prabawanto, 2021).

The deficiency in the development of concept formation skills of PMTs is reflected in their inability to generalize patterns and relationships obtained from exploration of mathematical situations, and to operationally formulate concepts in coherent mathematical language. Previous studies noted that the preservice teachers' understandings of geometry and measurement had not fully developed as evidenced by unproductive concept images, concept definitions, and lacking ability to present clearly their reasoning with geometric concepts. They found that the level of their understanding was comparable to that of primary graders or secondary school students (Browning et al., 2014).

The study findings on educational contextualization skills of PMTs point out learning gaps in connecting concepts to contexts, outlining contextualized learning activity and designing contextualized assessment strategy for a given topic. It is of common knowledge that teachers teach the way they are taught (Oleson & Hora, 2014); thus, this performance in contextualizing education may reflect the decontextualized teaching of mathematics practiced in teacher education institutions. When pre-service teachers are devoid of relevant contexts, they may accumulate isolated knowledge that operates only within mathematics. Consequently, students develop limited ability to remember ideas and knowledge that are divorced from meaningful contexts (Wang et al. 2022; Wotherspoon et al., 2015).

The significant correlations among mathematical communication, mathematical reasoning, concept formation and educational contextualization skills of PMTs imply the interconnectedness of these competences and the nonlinear progression of learning and skills development that TEIs may consider in implementing contextualized education. For instance, reasoning as the main feature of mathematics, cannot be separated from the activity of learning to include concept formation through inquiry and mathematical investigations. The consistent association between reasoning skills and concept formation verified in the study also echoes

earlier studies showing that reasoning and the ability to formulate generalizations are closely linked. Basically, correlation existed between scientific reasoning skills and mathematics achievement (Căprioară, 2015; Frosch & Simms, 2015; Hasanah et al., 2019; Tajudin & Chinnappan, 2015). Likewise, students who experienced difficulty in recognizing general statements also struggled in creating them as they were dependent on examples or specific cases to justify the truth of a statement (Stylianides & Stylianides, 2017; Sukirawan et al., 2018).

The predictive relationship between mathematical communication and concept formation highlights that the ability to operationally formulate mathematical knowledge stems from a profound comprehension of mathematical concepts and a command of mathematical language. Since concept formation is phenomenological (Hefendehl-Hebeker, 2016), mathematics learning becomes an empirical and object-oriented experience where contents and concepts learned are drawn by students from manipulation of tangible familiar objects in their respective meaningful realities (Pielsticker, 2021). This bridges concept formation and contextualization skills.

With concept formation skill as a significant predictor of educational contextualization ability, an increase in the level of ability in concept formation may produce a significant improvement in the ability of teachers to connect the teaching and learning of geometry across disciplines and other relevant contexts. This is because PMTs during concept formation connect their mathematical knowledge to the curriculum and pedagogy as the relevant and meaningful contexts (Buchbinder & McCrone, 2020; Burscheid & Struve, 2020; Pielsticker, 2021). The enhanced knowledge base developed then becomes contextualized and may present varied and relevant learning pathways that teachers may employ to address persistent drawbacks in mathematics teaching and learning (Xueli et al., 2022). However, it was noted that some PMTs entered mathematics training program with limited knowledge and experience to anchor mathematical concepts, their overall mathematical knowledge was limited and weak, and their prior abilities were focused primarily on manipulation of formulas. These findings, along with the study results, underscore the need for PMTs to have greater exposure to problems and learning experiences that promote conceptual understanding (Browning et al., 2014). The task for TEIs then is to initiate the PMTs into professional practice through which they are expected to embody what their future workplace requires, and to critically assess the interplay of content, cognitive and pedagogical competencies in teacher education curriculum.

Conclusions and Recommendations

The results of this study give credence to the reported development deficiencies in higher order cognitive abilities, and pedagogical competencies of pre-service and neophyte in-service mathematics teachers. It provides evidence on direct and consistent associations among mathematical communication, mathematical reasoning, concept formation, and instructional competencies of prospective teachers. This interplay of skills suggests that learning progression in both content and pedagogical knowledge entails integrative non-linear approaches in teacher education pre-service curriculum. Mathematical communication and reasoning are overarching skills that need to develop along with concept formation and educational contextualization skills of PMTs through contexts drawn from knowledge base and real-life experience in which these skills operate.

The research findings provide empirical data that may inform educators and curriculum planners in designing and implementing relevant pre-service training curricular content and pedagogy, and in crafting policies that detail integrative educational framework.

The study identifies some limitations that may open up future research directions. The study covered a limited number of participants from pilot testing to actual conduct of competency testing to generate substantial data that facilitate rigorous quantitative analysis and in-depth

review of research instruments. This is due to fewer college students taking mathematics in teacher education programs offered by colleges and universities within the region where the study was conducted.

As the present study was limited to the quantitative exploration of predictive relationships among the identified variables, a more structured related study employing additional variables and corresponding quantitative and qualitative analyses may be carried out to investigate the causality and paths of relationships.

Acknowledgements

The author acknowledges the participation of PMTs in the conduct of this study and the assistance provided by the Department of Science and Technology-Science Education Institute of the Republic of the Philippines as part of CBPSME scholarship package.

References

- Adams, A. (2010). Rehearsal or reorganization of two patterns of literacy strategy use in secondary mathematics class. *The Montana Mathematics Enthusiast*, 7(2), 371–390. <https://doi.org/10.54870/1551-3440.1195>
- Adams, T., & Lowery, R. (2007). An analysis of children's strategies for reading mathematics. *Reading & Writing Quarterly*, 23, 161–177. <http://dx.doi.org/10.1080/10573560601158479>
- Australian Curriculum. (2013). *Australian curriculum, assessment and reporting authority* (ACARA). Australian curriculum. <https://www.australiancurriculum.edu.au>
- Benneth, J., Lubben, F., & Hogarth, S. (2007). Bringing science to life: A synthesis of the research evidence on the effects of context-based and STS approaches to science teaching. *Science Education*, 91(3), 347–370. <https://doi.org/10.1002/sce.20186>
- Bergqvist, T., & Lithner, J. (2021). Mathematical reasoning in teachers' presentations. *Journal of Mathematical Behavior*, 31(2), 252–269. <https://doi.org/10.1016/j.jmathb.2011.12.002>
- Bottge, B. A. (1999). Effects of contextualized math instruction on problem solving of average and below-average achieving students. *The Journal of Special Education*, 33(2), 81–92. <https://doi.org/10.1177/002246699903300202>
- Bronkhorst, L., & Akkerman, S. (2006). At the boundary of schools' continuity and discontinuity in learning across contexts. *Educational Research Review*, 19, 18–35. <https://doi.org/10.1016/j.edurev.2016.04.001>
- Browning, C., Edson, A., Kimani, P., & Aslan-Tutak, F. (2014). Mathematical content knowledge for teaching of elementary mathematics: A focus on geometry and measurement. *The Mathematics Enthusiast*, 11(2). <https://doi.org/10.54870/1551-3440.1306>
- Buchbinder, O., & McCrone, S. (2020). Preservice teachers learning to teach proof through classroom implementation: Successes and challenges. *Journal of Mathematical Behavior*, 58. <https://doi.org/10.1016/j.jmathb.2020.100779>
- Burscheid, H. & Struve, H. (2020). *Mathematics education in reconstruction: A contribution to its foundation*. Springer. <https://doi.org/10.1007/978-3-658-29452-6>
- Căprioară, D. (2010). Problem solving - purpose and means of learning mathematics in school. *Procedia – Social and Behavioral Sciences*, 2(2), 1859–1864.
- Caswell, J., & Derek, J. (2017). Inquiry-based learning from the learner's point of view: A teacher candidate's success story. *Journal of Humanistic Mathematics*, 7, 161–186. <https://doi.org/10.5642/jhummath.201702.08>
- Chavarria-Arroyo, G., & Albanese, V. (2023). Contextualized mathematical problems: Perspectives of teachers about problem posing. *Education Sciences*, 13(1). <https://doi.org/10.3390/educsci13010006>
- Chu, S.K.W., Reynolds, R.B., Tavares, N.J., Notari, M., & Lee, C.W.Y. (2017). *Twenty-first century skills education in the US: An example of an inquiry-based game design learning approach. In: 21st century skills development through inquiry-based learning* (pp. 79–105). Springer. https://doi.org/10.1007/978-981-10-2481-8_5

- Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. SAGE Publications.
- Duran, M., & Dokme, I. (2016). The effect of the inquiry-based learning approach on students' critical-thinking skills. *Eurasia Journal of Mathematics, Science & Technology Education*, 12, 2887–2908. <https://doi.org/10.12973/eurasia.2016.02311a>
- Edwards, R. (2009). Introduction: "Life as a learning context?" In R. Edwards, G. Biesta, & M. Thorpe (Eds.). *Rethinking Contexts for Learning and Teaching. Communities, Activities and Networks*. Routledge.
- Eshetu, D., Atnaфу, M., & Woldemichael, M. (2022). The effectiveness of guided inquiry-based technology integration on pre-service mathematics teachers' understanding of plane geometry. *Journal of Pedagogical Research*, 6(4), 84–100. <https://doi.org/10.33902/JPR.202215241>
- Fraenkel, J., Wallen, N. & Hyun, H. (2012). *How to design and evaluate research in education* (5th Ed.). McGraw-Hill.
- Frederickson, S. R. (2019). The relationship between English language learners' mathematics problem solving strategies and the mathematics register. *Graduate Theses and Dissertations*. <https://scholarworks.uark.edu/etd/3413>
- Frosch, C.A., & Simms, V. (2015). Understanding the role of reasoning ability in mathematical achievement. *Euro Asian Pacific Joint Conference on Cognitive Science*. <https://www.semanticscholar.org/paper/Understanding-the-Role-of-Reasoning-ability-in-Frosch-Simms/cb677931f383ad4f23ab1d593fb66ff7b19f43b3>
- Fuja-Novitra, F., & Asrizal, F. (2021). Development of online-based inquiry learning model to improve 21st century skills of physics students in senior high school. *EURASIA Journal of Mathematics, Science and Technology Education*, 17(9). <https://doi.org/10.29333/ejmste/11152>
- Gebre, E., & Polman, J. (2020). From context to active contextualization: Fostering learner agency in contextualizing learning through science news reporting. *Learning, Culture and Social Interaction*, 24, Article 100374. <https://doi.org/10.1016/j.lcsi.2019.100374>
- Gilbert, J. (2006). On the nature of 'context' in chemical education. *International Journal of Science Education*, 28(9), 957–976. <https://doi.org/10.1080/09500690600702470>
- Gilje, O., & Erstad, O. (2017). Authenticity, agency and enterprise education: studying, learning in and out of school. *International Journal of Education Research*, 84, 58–67. <https://doi.org/10.1016/j.ijer.2016.05.012>
- Hasanah, S. I., Tafrilyanto, C. F., & Aini, Y. (2019). Mathematical reasoning: The characteristics of students' mathematical abilities in problem solving. *Journal of Physics: Conference Series*, 1188(1), Article 012057. <https://doi.org/10.1088/1742-6596/1188/1/012057>
- Hefendehl - Hebecker, L. (2016). Subject - matter didactics in German traditions. *Journal for Didactics of Mathematics*, 1, 11–31. <https://doi.org/10.1007/s13138-016-0103-7>
- Hofstein, A., Eilks, I., & Bybee, R. (2011). Societal issues and their importance for contemporary science education: A pedagogical justification and the state-of-the-art in Israel, Germany and the USA. *Instructional Journal of Science and Mathematics Education*, 9, 1459–1483. <https://doi.org/10.1007/s10763-010-9273-9>
- Hutapea, N. M., Saragih, S., & Sakur, S. (2019). Improving mathematical communication skills of SMP students through contextual learning. *Journal of Physics: Conference Series*, 1351(1), Article 012078. <https://doi.org/10.1088/1742-6596/1351/1/012067>
- Jader, J., Sidenvall, J., & Sumpter, L. (2017). Students' mathematical reasoning and beliefs in non-routine task solving. *International Journal of Science and Mathematics Education*, 15. <https://doi.org/10.1007/s10763-016-9712-3>
- Jenkins, D., Zeidenberg, M., & Kienzl, G. S. (2009). *Educational outcomes of I-BEST, Washington State Community and Technical College System's Integrated Basic Education and Skills Training Program: Findings from a multivariate analysis* (CCRC Working Paper No. 16). Columbia University, Teachers College, Community College Research Center. <https://ccrc.tc.columbia.edu/publications/i-best-multivariateanalysis.html>
- Jerrim, J., Oliver, M., & Sims, S. (2018). The relationship between inquiry-based teaching and students' achievement. New evidence from a longitudinal PISA study in England. *Learning and Instruction*, 61, 35–44. <https://doi.org/10.1016/j.learninstruc.2018.12.004>

- Kamid, Rusdi, K., Fitaloka, O., Basuki, F. & Anwar, K. (2020). Mathematical communication skills based on cognitive styles and gender. *International Journal of Evaluation and Research in Education*, 9(4). <https://doi.org/10.11591/ijere.v9i4.20497>
- Kleemans, T., & Segers, E. (2020). Linguistic precursors of advanced math growth in first language and second-language learners. *Research in Developmental Disabilities*, 103. <https://doi.org/10.1016/j.ridd.2020.103661>
- Keith, T. Z. (2019). *Multiple regression and beyond: An introduction to multiple regression and structural equation modeling*. Routledge. <https://doi.org/10.4324/9781315162348>
- Laursen, S. L., Hassi, M.-L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education* 45, 406–418. <https://doi.org/10.5951/jresmetheduc.45.4.0406>
- Lazar, M. K., Bean, R. M., & Van Horn, B. V. (1998). Linking the success of a basic skills program to workplace practices and productivity: An evaluation. *Journal of Adolescent & Adult Literacy*, 41(5), 352–362. <https://www.jstor.org/stable/40016938>
- Lithner, J. (2008). Students' mathematical reasoning in university textbook exercises. *Education Studies in Mathematics*, 52, 29–55. <https://doi.org/10.1023/A:1023683716659>
- Mallart, A., Font, V., & Diez, J. (2018). Case study on mathematics pre-service teachers' difficulties in problem solving. *Eurasia Journal of Mathematics, Science and Technology Education*, 14(4), 1465–1481. <https://doi.org/10.29333/ejmste/83682>
- Marzano, R. J. (2000). *Designing a new taxonomy of educational objectives*. Corwin Press. <https://eric.ed.gov/?id=ED447161>
- Mata-Pereira, J., & da Ponte, J. (2017). Enhancing students' mathematical reasoning in the classroom: teacher actions facilitating generalization and justification. *Educational Studies in Mathematics*, 96. <https://doi.org/10.1007/s10649-017-9773-4>
- Metz, D. (2013). Understanding contextual teaching in mathematics. *The MERN Journal*, 7(1), 54–64.
- Moely, A., & Connolly, J. (2020). The use of inquiry-based learning in public administration education: Challenges and opportunities in the context of internationalization. *Teaching Public Administration*, 39(3). <https://doi.org/10.1177/0144739420935>
- Mukuka, A., Balimutajjo, S., & Mutarutinya, V. (2023). Teacher efforts towards the development of mathematical reasoning skills. *Heliyon*, 9(4). <https://doi.org/10.1016/j.heliyon.2023.e14789>
- Mukuka, A., Shumba, O., Balimuttajjo, S., & Mutarutinya, V. (2019). An analysis of prospective teachers' mathematical reasoning on number concepts. *African Journal of Research in Mathematics, Science and Technology Education*, 15, 119–128. <https://doi.org/10.4314/ajesms.v15i2.10>
- Nashon, S., & Anderson, D. (2013). Interpreting student views of learning experiences in a contextualized science discourse in Kenya. *Journal of Research in Science Teaching*, 50(4), 381–407. <https://doi.org/10.1002/tea.21078>
- Nilson, P., & Ryve A. (2014). The nature and role of common ground in the learning of mathematics in small-group discussions. *Scandinavian Journal of Educational Research*, 58(5). <https://doi.org/10.1080/00313831.2013.821087>
- Nikitina, S. (2006). Three strategies for interdisciplinary teaching: Contextualizing, conceptualizing, and problem-centering. *Journal of Curriculum Studies*, 38(2), 251–271. <https://doi.org/10.1080/00220270500422632>
- Novianda, D., & Prabawanto, S. (2021). Analysis of students' mathematical reasoning ability in geometry through distance learning. *Journal of Physics: Conference Series*, 1882, Article 012085. <https://doi.org/10.1088/1742-6596/1882/1/012085>
- Oleson, A., & Hora, M. (2014). Teaching the way they were taught? Revisiting the sources of teaching knowledge and the role of prior experience in shaping faculty teaching practices. *Higher Education*, 68, 29–45. <https://doi.org/10.1007/s10734-013-9678-9>
- Palinussa, A.L., Molle, J.S., & Gaspersz, M. (2021). Realistic mathematics education: mathematical reasoning and communication skills in rural contexts. *International Journal of Evaluation and Research in Education*, 10 (2), 522–534.
- Parr, B. A., Edwards, M. C., & Leising, J. G. (2008). Does a curriculum integration intervention to improve the mathematics achievement of students diminish their acquisition of technical competence? An experimental study in agricultural mechanics. *Journal of Agricultural Education*, 49(1), 61–71. <https://doi.org/10.5032/jae.2008.01061>

- Perin, D. (2011). Facilitating student learning through contextualization: A review of evidence. *Community College Review*, 39, 268–295. <https://doi.org/10.1177/0091552111416227>
- Picardial, M., & Sanchez, J. (2022). Effectiveness of contextualization in science instruction to enhance science literacy in the Philippines: A meta-analysis. *International Journal of Learning, Teaching and Educational Research*, 21, 140–156. <https://doi.org/10.26803/ijlter.21.1.9>
- Pielsticker, F. (2021). Concept formation processes regarding height and base in triangles. *International Electronic Journal of Mathematics Education*, 16(2). <https://doi.org/10.29333/iejme/10891>
- Preston, L., Harvie, K., & Wallace, H. (2015). Inquiry-based learning in teacher education: A primary humanities example. *Australian Journal of Teacher Education*, 40(12), 73–85. <https://doi.org/10.14221/ajte.2015v40n12.6>
- Rao, A. (2006). Cognition and motor skills. In *Hand Function in the Child* (2nd Ed). Henderson Anne & Pehoski Charlene (Eds). Mosby. <https://doi.org/10.1016/B978-0-323-03186-8.X5001-1>
- Roslan, A., Phang, F., Puspanathan, J., & Nawi, N. (2023). Challenges in implementing inquiry-based learning in physics classroom. *AIP Conference Proceedings*, 2569(1), Article 050003. <https://doi.org/10.1063/5.0117509>
- Rustam, A., & Ramlan, A.M. (2017). Analysis of mathematical communication skills of junior high school students of Coastal Kolaka. *Journal of Mathematics Education*, 2(2), 45–50.
- Schunk, D. H. (2012). *Learning theories: An educational perspective* (6th Ed). Pearson.
- Silzeth, K., & Erstad, O. (2018). Connecting to the outside: Cultural resources teachers use when contextualizing instruction. *Learning, Culture and Social Interaction*, 17, 56–68. <https://doi.org/10.1016/j.lcsi.2017.12.002>
- Sukirawan, Darhim, & Herman, T. (2018). Analysis of students' mathematical reasoning. *Journal of Physics: Conference Series*, 948(1). <https://doi.org/10.1088/1742-6596/948/1/012036>
- Supriyanto, J., Suparman, & Hairun, Y. (2020). Design of worksheets for RME model to improve mathematical communication. *Universal Journal of Educational Research*, 8(4), 1363–1371.
- Stevens, J. (1996). *Applied multivariate statistics for the social sciences* (3rd ed.). Lawrence Erlbaum Associates.
- Stylianides, G. J., & Stylianides, A. J. (2017). Research-based interventions in the area of proof: The past, the present, and the future. *Educational Studies in Mathematics*, 96, 119–127. <https://doi.org/10.1007/s10649-017-9782-3>
- Tabachnick, B. G., & Fidell, L. S. (2007). *Using multivariate statistics* (9th ed.). Allyn & Bacon/Pearson Education.
- Tajudin, N., & Chinnappan, M. (2015). Exploring relationship between scientific reasoning skills and mathematics problem solving. In M. Marshman V. Geiger & A. Bennison (Eds.). *Mathematics education in the margins*, Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia. <https://files.eric.ed.gov/fulltext/ED572516.pdf>
- Tall, D. (2013). *How humans learn to think mathematically. Exploring the three worlds of mathematics*. Cambridge University Press. <https://doi.org/10.1017/CBO9781139565202>
- Tatto, M. (2020). Teacher education development study-mathematics (TEDS-M). In: Lerman, S. (Eds.), *Encyclopedia of Mathematics Education*. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_151
- Tatto, M. T. (2013). *Teacher Education and Development Study in Mathematics (TEDS-M): Technical Report*. International Association for Educational Achievement (IEA). https://www.iea.nl/sites/default/files/2019-05/TEDS-M_technical_report.pdf
- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, J., Rodriguez, M., Ingvarson, L., Reckase, M., & Rowley, G. (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M)*. International Association for the Evaluation of Educational Achievement (IEA). <https://eric.ed.gov/?id=ED542380>
- Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Peck, R., & Rowley, G. (2008). *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics. Conceptual Framework*. Teacher Education and Development International Study Center, College of Education. [https://www.semanticscholar.org/paper/Teacher-education-and-development-study-in-\(TEDS-M\)-Tatto-Schwille/d4628dc1806eac6f4fd612dd2c34d43552da88d5](https://www.semanticscholar.org/paper/Teacher-education-and-development-study-in-(TEDS-M)-Tatto-Schwille/d4628dc1806eac6f4fd612dd2c34d43552da88d5)

Judel V. PROTACIO. Predictors of concept formation and educational contextualization competencies in geometry of pre-service mathematics teachers

- Teig, N. (2021). Inquiry in science education. In T. Nilsen, A. Stancel Piatak, & J. E. Gustafsson (Eds.). *International handbook of comparative large-scale studies in education*. Springer Open. https://doi.org/10.1007/978-3-030-38298-8_62-1
- Teig, N., Scherer, R., & Nilsen, T. (2018). More isn't always better: The curvilinear relationship between inquiry-based teaching and student achievement in science. *Learning and Instruction*, 56, 20–29. <https://doi.org/10.1016/j.learninstruc.2018.02.006>
- Tong, D., Uyen, B., & Quoc, N. (2021). The improvement of 10th-grade students' mathematical communication skills through learning ellipse topics. *Heliyon*, 7. <https://doi.org/10.1016/j.heliyon.2021.e8282>
- Valdez, J., & Bungihan, M. (2017) Problem-based learning approach enhances the problem-solving skills in chemistry of high school students. *Journal of Technology and Science Education*, 9(3), 282–294. <https://doi.org/10.3926/jotse.631>
- Vygotsky, L. S. (1986). *Thought and language* (A. Kozulin, Ed. & Trans.). MIT Press. (Original work published 1934).
- Wang, X., Lee, Y., Zhu, X., & Ozdemir, A. (2022). Exploring the relationship between community college students' exposure to math contextualization and educational outcomes. *Research in Higher Education*, 63(2), 309–336. <https://doi.org/10.1007/s11162-021-09644-w>
- Zulkarnain, I., Kusumawati, E., & Mawaddah, S. (2021). Mathematical communication skills of students in mathematics learning using discovery learning model. *Journal of Physics: Conference Series*, 1760. <https://doi.org/10.1088/1742-6596/1760/1/012045>

Received: February 27, 2025 Revised: March 28, 2025

Accepted: April 10, 2025

Cite as: Protacio, J. V. (2025). Predictors of concept formation and educational contextualization competencies in geometry of pre-service mathematics teachers. *Problems of Education in the 21st Century*, 83(2), 218–236. <https://doi.org/10.33225/pec/25.83.218>

Judel V. Protacio

PhD, Professor, College of Education, Arts and Sciences, Capiz State University, Philippines.

E-mail: jvprotacio@capsu.edu.ph

ORCID: <https://orcid.org/0000-0003-0468-4166>